GCSE Mathematics (1MA1) – Aiming for Grade 9 2H

Student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for marking scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

The following table shows the marks scored on average at certain grades on similar questions from live exams.

For example: A student who achieved a Grade 9 on similar questions from either the Summer 2023 or November 2023 exam sittings achieved on average 28.4 marks from these questions**.**

Grade	**9**	**8**	**7**	**6**	**5**	**4**	**3**
Mark	28.4	19.4	12.6	8.3	4.6	2.1	1

Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The incomplete table and the incomplete histogram give information about the times taken by some students to run a race.

Freq. density

Time (t seconds)	Frequency
$10 < t \leq 12$	3
$12 < t \leq 16$	10
$16 < t \leq 19$	15
$19 < t \leq 21$	9
$21 < t \leq 26$	7

1 mark

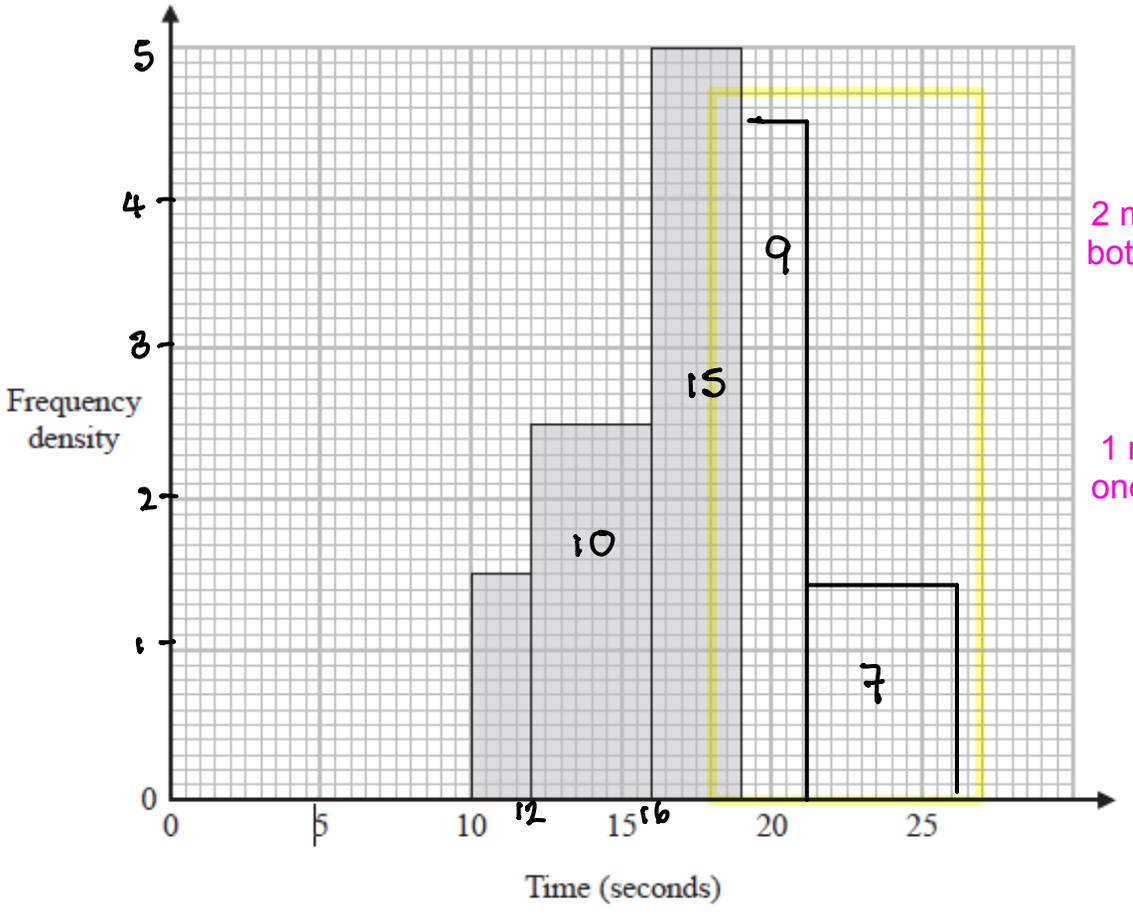
$1.5 \times 2 = 3$

$10 \div 4 = 2.5$

$15 \div 3 = 5$

$9 \div 2 = 4.5$

$7 \div 5 = 1.4$



2 marks for both correct bars

1 mark for one correct bar

None of these students had a time for the race such that $t < 10$ or $t > 26$

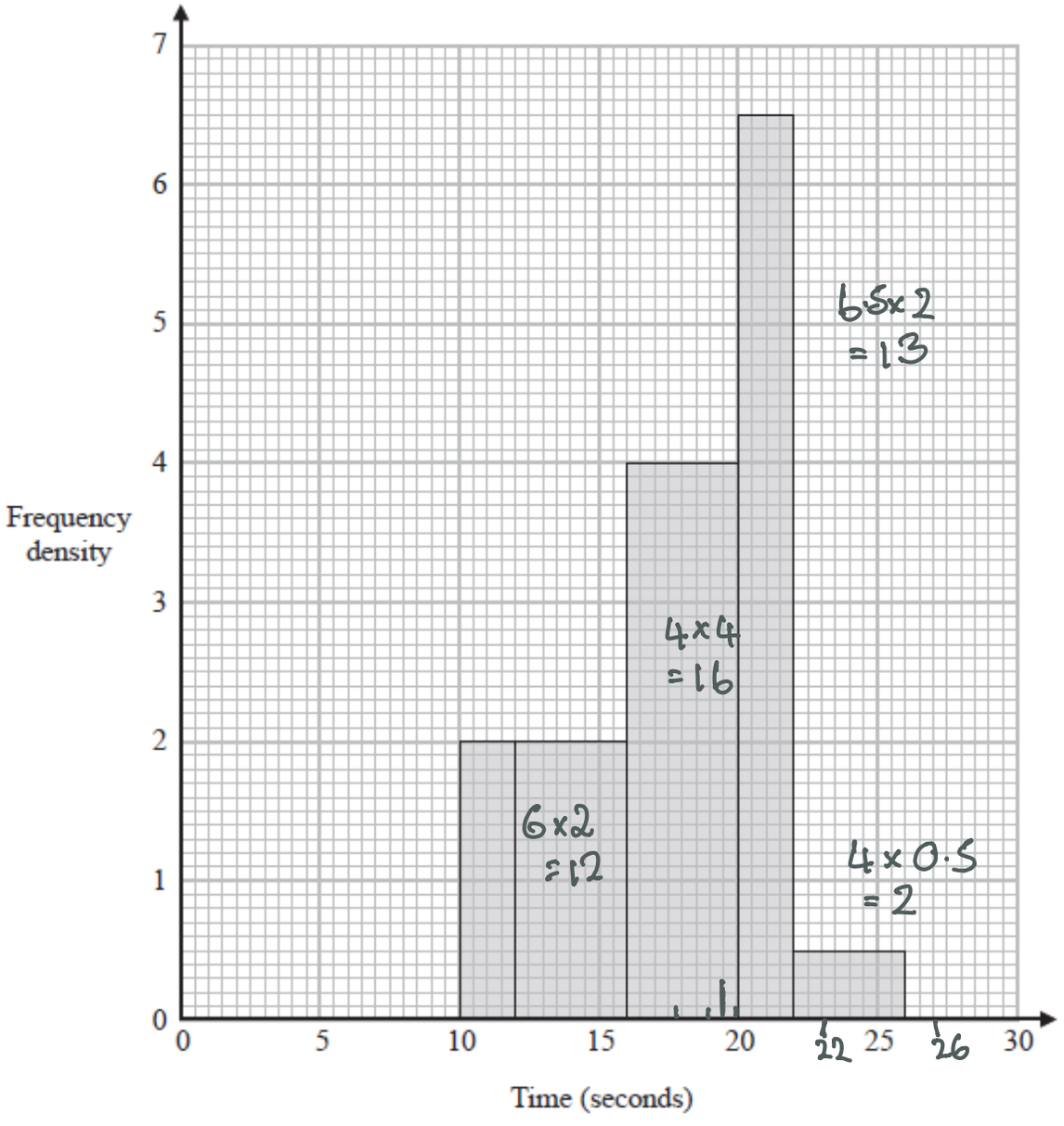
(*a*) Use the histogram to complete the table.

(1)

(*b*) Use the table to complete the histogram.

(2)

The histogram below gives information about the times taken by 43 students to run a different race.



(*c*) Work out an estimate for the median of the times taken by these 43 students to run the race.

1 mark

Total = 12 + 16 + 13 + 2 = 43

median = $\frac{43+1}{2}$ = 22nd

1 mark

22 - 12 = 10

Final mark

18.5 ..... seconds

(3)

(Total for Question 1 is 6 marks)

2 A biased dice is thrown 60 times.
The table shows information about the number that the dice lands on each time.

Number on dice	1	2	3	4	5	6
Frequency	12	7	8	9	9	15

Gethin throws the dice twice.

(*a*) Work out an estimate for the probability that the dice will land on 6 both times.

1 mark $P(6) = \frac{15}{60} = \frac{1}{4}$

$P(6,6) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

1 mark

Final mark

$\frac{1}{16}$ $\left(\frac{225}{3600}\right)$

(3)

Sally is going to throw the same dice *n* times and record the number it lands on each time.
She will use her results to work out a more reliable estimate for the probability in part (*a*).

(*b*) What can you say about the value of *n*?

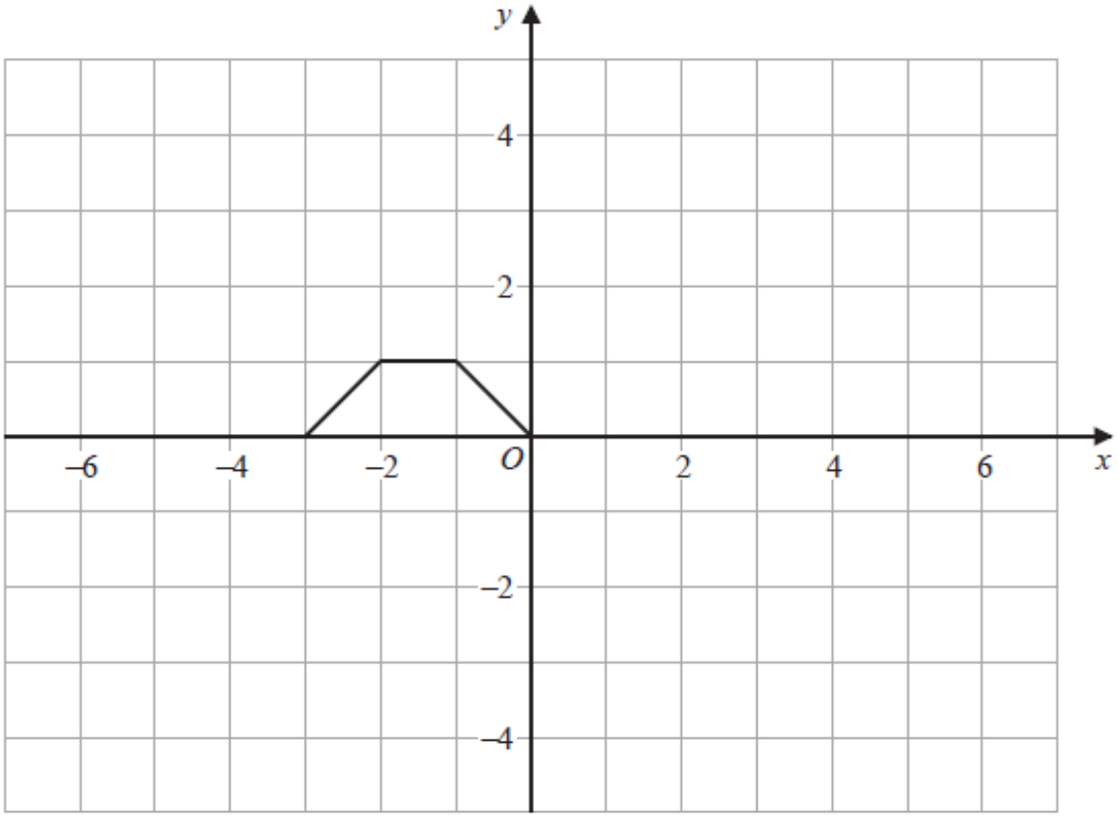
The greater the value of n, the more reliable her results will be so n should be greater than 60

1 mark

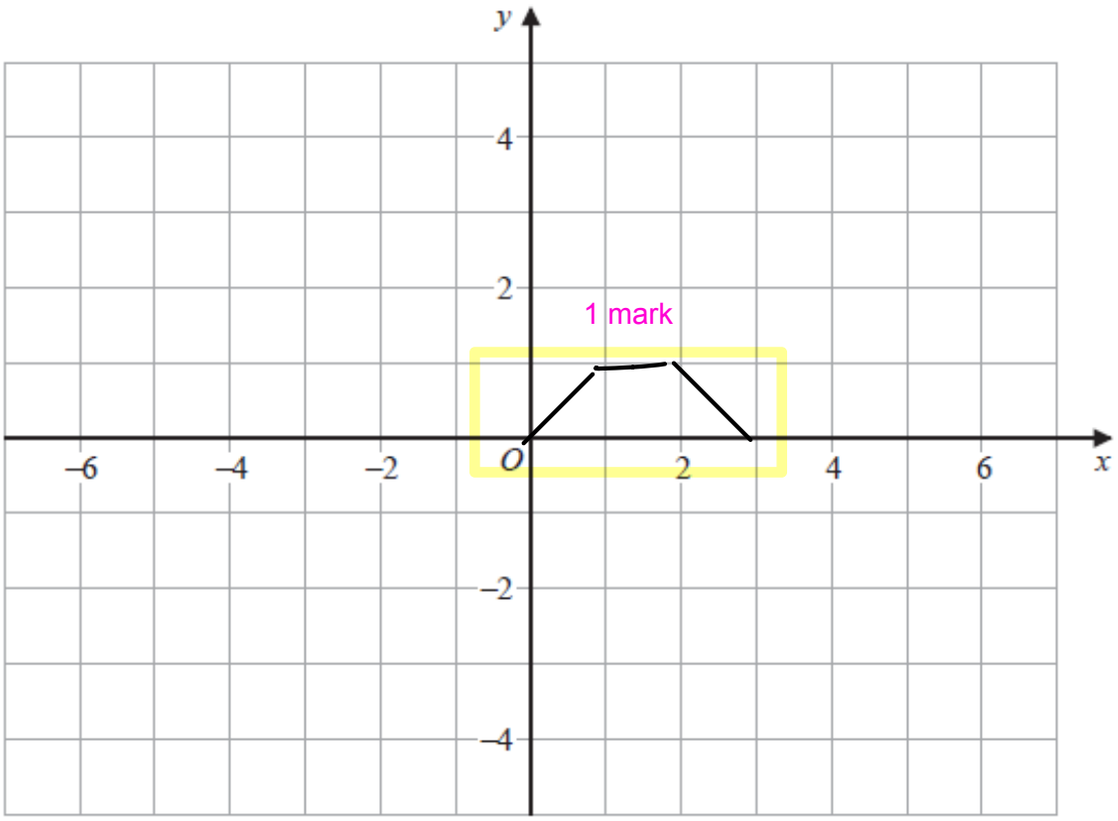
(1)

(Total for Question 2 is 4 marks)

3 Here is the graph of $y = f(x)$

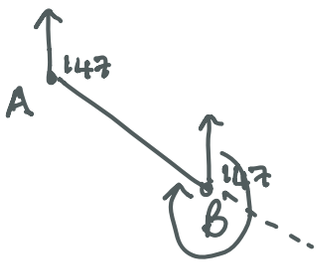


On the grid below, draw the graph of $y = f(-x)$



(Total for Question 3 is 1 mark)

***4** The bearing of port B from port A is 147°
Work out the bearing of port A from port B.



1 mark

147 + 180
= 327

Final mark

327

..... °

(Total for Question 4 is 2 marks)

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5 $2a : 5c = 6 : 25$
$4b : 7c = 20 : 21$

Show that $a + b : b + c = 17 : 20$

2a : 5c
6 : 25

4b : 7c
20 : 21

a : c
3 : 5
9 : 15
×3

1 mark

b : c
5 : 3
25 15
×5

a : b : c

1 mark

9 : 25 : 15

Final mark

a + b = 34 b + c = 40

34 : 40

= 17 : 20 as required.

(Total for Question 5 is 3 marks)

6 Write $\frac{14}{3x-21}+\left[(x+4)\div\frac{2x^2-6x-56}{2x+3}\right]$ in the form $\frac{ax+b}{cx+d}$ where a, b, c and d are integers.

$2x^2-6x-56 = (x+4)(2x-14)$ 1 mark

$\frac{14}{3x-21} + (x+4) \times \frac{2x+3}{2x^2-6x-56}$ 1 mark

$= \frac{14}{3x-21} + \frac{(x+4)(2x+3)}{(x+4)(2x-14)}$

$= \frac{14(2x-14)+(2x+3)(3x-21)}{(3x-21)(2x-14)}$ 1 mark

$= \frac{28x-196+6x^2-42x+9x-63}{(3x-21)2(x-7)}$

$= \frac{6x^2-5x-259}{2(3x-21)(x-7)}$

$= \frac{(6x+37)(x-7)}{2(3x-21)(x-7)}$

$= \frac{6x+37}{6x-42}$

Final mark

$\frac{6x+37}{6x-42}$

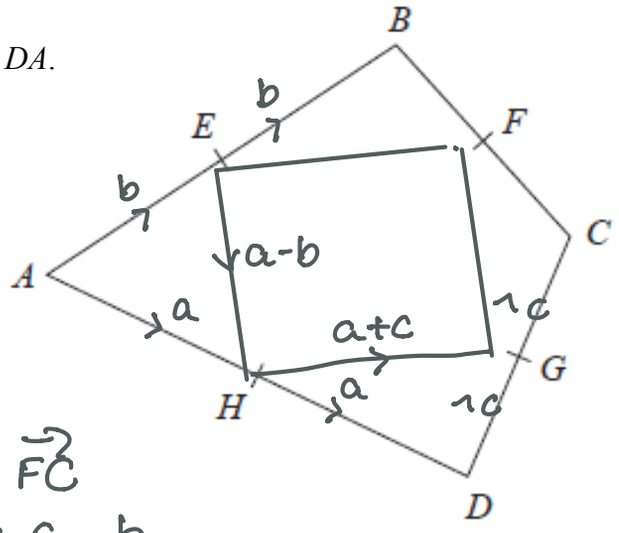
.....

(Total for Question 6 is 4 marks)

7 *ABCD* is a quadrilateral.
E, *F*, *G* and *H* are the midpoints of *AB*, *BC*, *CD* and *DA*.

$\overrightarrow{AH} = \mathbf{a}$ $\overrightarrow{AE} = \mathbf{b}$ $\overrightarrow{DG} = \mathbf{c}$

Prove, using vectors, that *EFGH* is a parallelogram.



1 mark

$\overrightarrow{BC} = 2a + 2c - 2b$ so $\overrightarrow{BF} = \overrightarrow{FC}$
$= a + c - b$

$\overrightarrow{EH} = a - b$ ← 1 mark for either of these → $\overrightarrow{HG} = a + c$

$\overrightarrow{FG} = \overrightarrow{FC} + \overrightarrow{CG}$
$= a + c - b - c$
$= a - b$

← 1 mark for either of these →

$\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF}$
$= b + a + c - b$
$= a + c$

so $\overrightarrow{EH} = \overrightarrow{FG}$ so $\overrightarrow{HG} = \overrightarrow{EF}$

$\therefore$ EFGH is a parallelogram

Final mark

(Total for Question 7 is 4 marks)

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8 Show that the equation $x^3 + 2x - 6 = 0$ has a solution between $x = 1$ and $x = 2$

$x = 1$ $1^3 + 2 \times 1 - 6 = 1 + 2 - 6 = -3$

1 mark for either of these

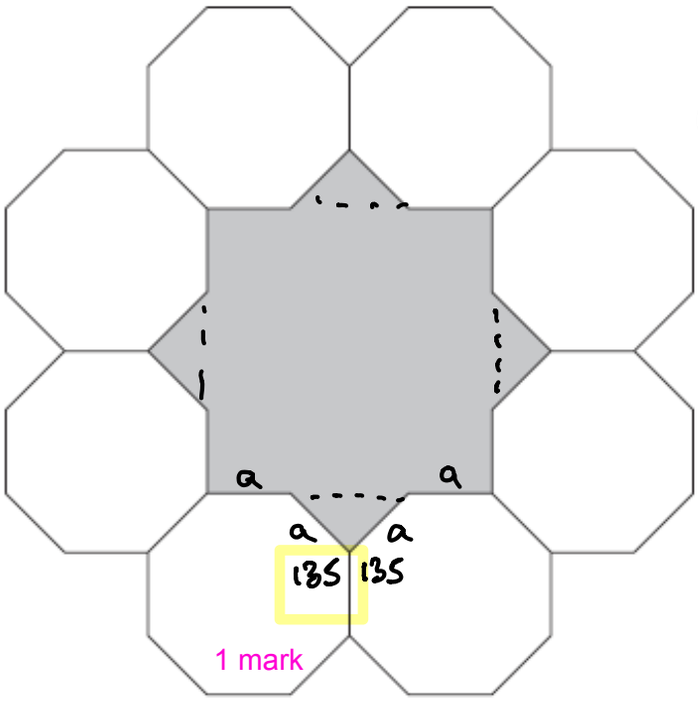
$x = 2$ $2^3 + 2 \times 2 - 6 = 8 + 4 - 6 = 6$

Final mark

since there is a change in sign a solution must lie between $x = 1$ and $x = 2$

(Total for Question 8 is 2 marks)

9 The diagram shows 8 identical regular octagons joined to enclose a shaded shape.



1 mark

(1) $1080 \div 8 = 135$

$360 - 270 = 90$

so right angle △

Each octagon has sides of length a.

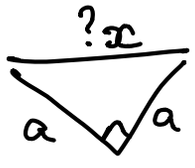
Find, in terms of a, an expression for the area of the shaded shape.

Give your answer in the form $p\left(2+\sqrt{2}\right)a^2$ where p is an integer.

You must show all your working.

1 mark

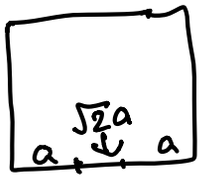
(2)



$x^2 = 2a^2$

$x = \sqrt{2}a$

area of $\frac{1}{2}$ △ $= \frac{1}{2} a \times a = \frac{1}{2}a^2$



4 OF THESE ! $= 2a^2$

area of square

$= (2a + \sqrt{2}a)(2a + \sqrt{2}a)$

$= 4a^2 + 4\sqrt{2}a^2 + 2a^2$

$= 6a^2 + 4\sqrt{2}a^2$

1 mark for side length of the square

1 mark

Total $= 6a^2 + 4\sqrt{2}a^2 + 2a^2 = 8a^2 + 4\sqrt{2}a^2$

Final mark $= 4(2+\sqrt{2})a^2$ as required

(Total for Question 9 is 5 marks)

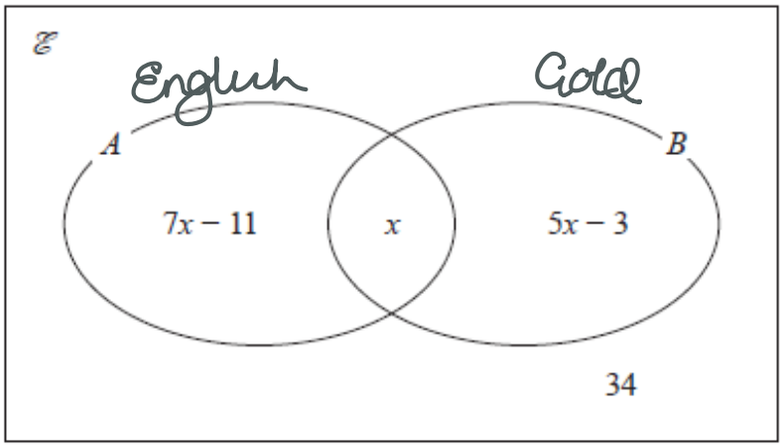
10 Vicky has a collection of medals.

The Venn diagram gives information about the number of medals in her collection where

$\mathscr{E}$ = {all medals}

A = {English medals}

B = {gold medals}



Vicky is going to take at random a medal from her collection.

Given that the medal is gold, the probability that the medal is English is $\frac{2}{11}$

Work out the number of medals in Vicky's collection.

$\mathscr{E} = 7x - 11 + x + 5x - 3 + 34$

$= 13x + 20$

Given that the medal is gold $x + 5x - 3 = 6x - 3$

$\frac{x}{6x-3} = \frac{2}{11}$ 1 mark

$11x = 2(6x - 3)$

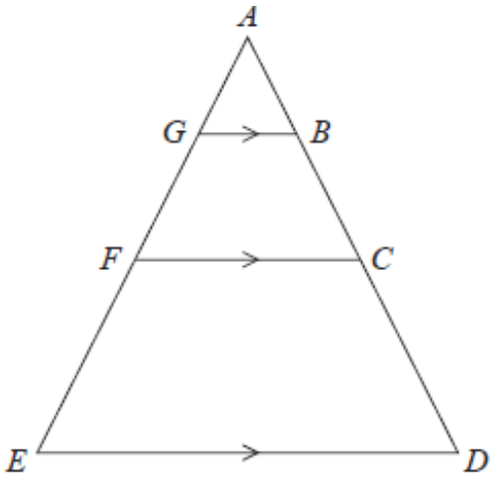
$11x = 12x - 6$ 1 mark

$x = 6$ so medals $= 13 \times 6 + 20$ 1 mark

98 Final mark

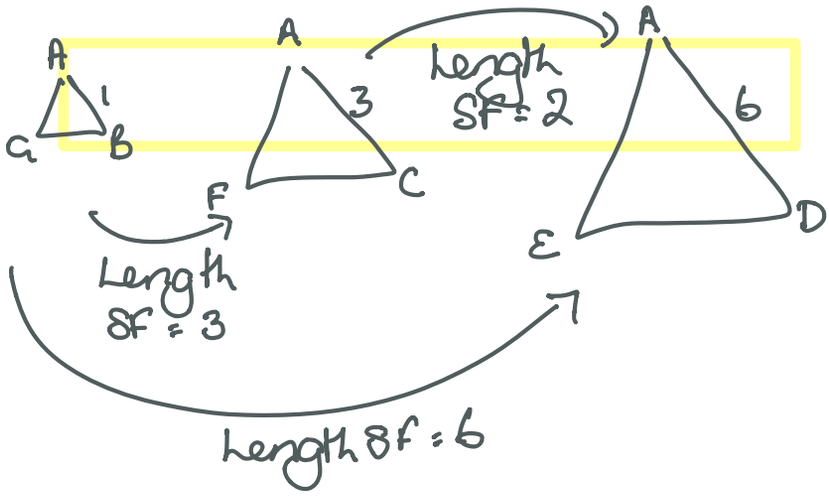
(Total for Question 10 is 4 marks)

11 Here are three similar triangles, *ABG*, *ACF* and *ADE*.



ABCD and *AGFE* are straight lines.
AB : *BC* : *CD* = 1 : 2 : 3
Show that

area of *ABG* : area of *BCFG* : area of *CDEF* = 1 : 8 : 27



1 mark

BCFG = △ACF − △ABG
area SF = $3^2 = 9$
= 9ABG − ABG
= 8ABG

CDEF = △ADE − △ACF
= 36ABG − 9ABG
= 27ABG

1 mark for either of these

∴ ratio of area 1 : 8 : 27

Final mark

(Total for Question 11 is 3 marks)

12 There are only blue pens and red pens in a box.

The number of blue pens is four times the number of red pens.
Rita takes at random one pen from the box.

She records the colour of the pen and then replaces it in the box.
Rita does this n times, where $n \geq 2$

Write down an expression, in terms of n, for the probability that Rita gets a blue pen at least once and a red pen at least once.

B
4R

R
R

$P(B) = \frac{4R}{5R} = \frac{4}{5}$

$P(R) = \frac{R}{5R} = \frac{1}{5}$

$P(\text{all B}) = \left(\frac{4}{5}\right)^n$

$P(\text{all R}) = \left(\frac{1}{5}\right)^n$

1 mark for either of these

Final mark

$1 - \left(\frac{4}{5}\right)^n - \left(\frac{1}{5}\right)^n$

.....

(Total for Question 12 is 2 marks)

TOTAL FOR PAPER IS 36 MARKS